PEDAGOGICAL WORK OF
MIHAILO PETROVIĆ ALAS (1868–1943)

On the occasion of Mihailo Petrović's 150th anniversary

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Abstract. The aim of this text is to look back on the creativity of Mihailo Petrović Alas, a well-known Serbian mathematician and the founder of the Belgrade Mathematical School, on the occasion of his 150th anniversary. The text deals with his contributions to mathematical teaching, as well as the influence of his pedagogical ideas on the development of mathematics in Serbia until these days.

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> *"Only ignorant and unreasonable people think that the past is dead and forever separated from the present by an impassable wall. On the contrary, the truth is that everything man used to think, feel and do is inextricably woven into what we think, feel and do toady. Bringing the light of scientific truth in the events from the past means serving the present time".*
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> Ivo Andrić

1. Introduction

This is the year when we celebrate the 150th anniversary of Mihailo Petrović Alas, probably one of a hundred of the most interesting and the most versatile personalities in the history of our people. And certainly one of the most significant personalities in Serbian science and culture at the end of the 19th and the first half of the 20th century, but also one of the most interesting and the most productive creators in Serbian mathematical science in general.

The aim of this text is to look back to the creativity of Mihailo Petrović in the field of mathematics and to analyze his contribution to mathematics teaching, as well as the influence of his pedagogical ideas on the development of mathematics in Serbia until these days.

The text deals with the life and work of Mihailo Petrović, Belgrade School of Mathematics, doctoral students of his, as well as his numerous activities related to the upgrading of mathematics teaching in Serbian schools and universities.

2. Biography of Mihailo Petrović

Mihailo Petrović was born in Belgrade on the 24th April 1868 (the 8th May in the Gregorian calendar) of father Nikodim, a professor of theology, and mother Milica. Since he lost his father at an early age, his upbringing and education was highly influenced by his mother's father, archpriest Novica Lazarević, thanks to whom Mihailo Petrović grew to love books and became a curious young man directed towards knowledge and science. At the age of six, in 1874, Mihailo Petrović started going to elementary school, and in 1878 he enrolled in the First Belgrade Grammar School which was then in Captain Miša's Mansion – the building where Mihailo Petrović would spend 55 years (interrupted only during his studies in Paris).

Brought up as a versatile person, Mihailo Petrović started playing the violin in 1880 and later became a true master in performing folk and urban folk music. He was the founder of the musical society "Suz" in 1892 with which he performed until the end of his life and with which he recorded series of old and forgotten folk melodies for Radio Belgrade during a period of thirty years. Some time later, in 1882, Mihailo Petrović started being a fishing trade apprentice with Đura Pupa, the Danube alas (fisherman), and was later certified in the fisherman trade and became a master river fisherman. That was how he got his famous nickname Mika-Alas.

He finished the First Belgrade Grammar School in 1884 and sat for the maturity exam with excellent grades. In the same year he enrolled in the natural science section of the Faculty of Philosophy of the Grand School in Belgrade, and in 1886 he wrote his first mathematical work – a term paper "On a Modification of Graeffe's Method for Solving Equations of Higher Order". He was awarded the Order of St. Sava at the Technical Faculty of Grand School in 1889 for his work in the field of computing machines, the topic of which was area measurement.

Mihailo Petrović finished his studies at Grand School in July 1889. In the same year he left for Paris where he was preparing for the entrance examination to l'École Normale Supérieure for a year. In the following year, 1890, he got another Order of St. Sava for his work in the field of analytical geometry. Having sat for the written part of the exam first, and then the oral part of the entrance examination in July, he was accepted at l'École Normale Supérieure as one of rare foreigners. Mihailo Petrović worked hard in Paris, and in 1892 he got a state scholarship and became a cadet of the Kingdom of Serbia. In the same year he graduated in mathematical sciences, and in 1893 he graduated in physical sciences. In Paris, Mihailo Petrović attended the courses held by famous French mathematicians of that time – professors Poincaré, Picard, Painlevé and Darboux.

On the 29th June 1894 Mihailo Petrović defended his doctoral dissertation at Paris University. The title of his dissertation was "On Zeros and Infinities of the Integrals of Algebraic Differential Equations". His doctoral dissertation commission was made up of Hermite (president), Picard and Painlevé (examiners). By defending his doctoral dissertation Mihailo Petrović was awarded the title of the Doctor of Philosophy in Mathematical Sciences. Only few months later, according to the Decree P. No. 863 dated 22nd October 1894, Mihailo Petrović was appointed

The first professors of Belgrade University, Mihailo Petrović is sitting to the right

as a full-time mathematics professor at the Faculty of Philosophy of Grand School in Belgrade. He worked at Grand School, which became the University of Belgrade in 1905, for 44 years, until he retired in 1938.

3. Mihailo Petrović's scientific work

The scientific work of Mihailo Petrović was connected mainly to two institutions: Belgrade University and Serbian Royal Academy (later renamed as Serbian Academy of Sciences and Arts), whose correspondent member he became in 1897, and full member in 1899. He delivered his introductory speech at Serbian Royal Academy in 1900, on the occasion of which he spoke about mathematical phenomenology (see: Mihailo Petrović: "On mathematical theory of sample activity" [2], Vol. 6, pp. 222–265), i.e. about the analogies of different natural processes (see: [2], Vol. 6: Mathematical Phenomenology).

The scientific work of Mihailo Petrović includes the problems of theory of functions, differential equations, problems from algebra, number theory, theory of probability and other mathematical fields. He published over 400 mathematical papers (328 mathematical and 74 applied mathematics articles) and approximately 100 non-mathematical papers. He had good ideas and problem solutions. His articles were the result of powerful mathematical intuition, they were original and abundant with open questions. He did not deal with further transformations of his ideas, generalisations and similar procedures.

Mihailo Petrović published his papers in 30 foreign journals and over 30 home

magazines and journals. He loved travelling and often participated in international congresses and gatherings of mathematicians. From Paris in 1900, via Toronto (1924), Zürich (1932), Prague (1934), Bucharest (1937) ... he took part in about 40 mathematical congresses, presiding to several scientific clubs and being the vice president of several congresses. He was the member of many European associations of mathematicians and a correspondent member of many European academies of sciences.

Different opinions on the mathematical work of Mihailo Petrović can be heard today, ranging from excitement to criticism. The critics do not diminish the significance of the great scientific work and contribution of Mihalo Petrović in any way, which is seen in the review of Dragan Trifunović, probably the best authority on Mihailo Petrović's life and work [1]: "Our research on Petrović's works states his failures in addition to his obvious success and acknowledged results. He worked alone at the Faculty of Philosophy in Belgrade for years (1894–1921) and that can make allowances for those failures. He had brought a lot of manners and attitudes of French mathematicians from Paris to his homeland, which were, obviously, very specific. Thus, there were no geometry, no new language of vectors and tensors, and not even a trace of linear algebra and matrices in Petrović's work. We have reached the conclusion that our professor used to make a whole 'scientific dispute' out of a single task or some small, insignificant problem, which could not have happened to European mathematicians. His methodology and the presentation of a dispute were completely in the manner of French texts of the second half of the 19th century. Such was his style of writing until the end of his life. Contemporary problems did not exist in his work. He never changed! Even though he worked and created until the middle of our century, he never separated himself from the contents of the 19th century. Personal attitudes to the hypothesis of continuum, studies on properties of different abstract spaces, set theory and other works of Cantor, "dangerous" Kronecker, Dedekind and many other more significant creators cannot be found in his works. Theory of measure and integration was completely left out. There were almost no limit processes. Riemann's manifolds were not mentioned at all, nor were there the results of Hilbert and open problems in the work of Mihailo Petrović".

M. Petrović, 1905

4. Pedagogical work of Mihailo Petrović

Mihailo Petrović's pedagogical work is an unavoidable part of his fruitful and fascinating creativity, and it left significant traces in his days, but it has also reached our days through those who continued his work.

Upon his arrival from Paris in 1894, Mihailo Petrović started organizing work on the popularization of mathematics, upgrading mathematics teaching, establi-

shing teaching staff and educating the mathematics teaching staff who were working in grammar schools and vocational schools of Serbia at that time. The result of his efforts are known today by a unique name – Belgrade School of Mathematics.

Certainly the most significant contribution of Mihailo Petrović in this sphere were his numerous students and associates who accepted their professor's scientific method and enriched it with new ideas and results.

During the first several decades of the 20th century at Belgrade University, Mihailo Petrović was the supervisor of ten doctoral students of mathematical sciences. The following table presents all his doctoral students and their results.

No.	The name of a PhD student	The year in which dissertation was defended	The number of PhD students	The number of scientific heirs
1	Mladen Berić	1912		
2	Sima Marković	1913		
3	Tadija Pejović	1923	17	168
4	Radivoj Kašanin	1924	2	16
5	Jovan Karamata	1926	12	405
6	Miloš Radojčić	1928		
7	Dragoslav Mitrinović	1933	33	120
8	Danilo Mihnjević	1934		
9	Konstantin Orlov	1934	9	76
10	Dragoljub Marković	1938	1	1

The source of data for the above given table is Mathematics Genealogy Project which was jointly started by the Department of Mathematics North Dakota State University and American Mathematical Society. This open project is available via `http://www.genealogy.ams.org` and it follows the scientific heirs of the most famous world mathematicians. At present there are 893 scientific heirs of Mihailo Petrović, 10 of whom were his doctoral students, 74 of whom were doctoral students of his doctoral students, etc. (the figures date from 12th February 2018).

What is even more important than those statistical data is the fact that the scientific heirs of Mihailo Petrović's (as well as those of his mentor-colleague Jacques Hadamard[1] whose scientific heir was Đuro Kurepa among others), as well as the heirs of their students, once had and still have a significant influence on the development of scientific thinking and the teaching of mathematics in Serbia.

A separate article could be written about what Mihailo Petrović's doctoral

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[1]They both defended their doctoral dissertation with Emil Picard – J. Hadamard in 1892, and M. Petrović in 1894.

students and their scientific heirs meant to the development of mathematics in Serbia. The fact is that Tadija Pejović, Radivoje Kašanin, Miloš Radojčić, Drago-slav Mitrinović, Konstantin Orlov and Dragoljub Marković left an indelible trace in Serbian mathematics of the 20th century, and most university professors in Serbia today are their scientific heirs. Jovan Karamata (1903–1967) made a special contribution not only to ours, but also to global mathematical thinking, and he is probably our most cited mathematician of all times. He was the author of the theory on regularly varying functions that has a big role in the modern probability theory. He was also the author of a series of works, particularly dealing with mathematical analysis, which, even today, several decades since being published, serve as a starting point in mathematical research, which is not a rule in this science, but a rare and honourable exception.

In his monography [1], Dragan Trifunović wrote that Belgrade Mathematical School started "working" on the 19th May 1912 when Mihailo Petrović addressed the Council of the Faculty of Philosophy, requesting that his first doctoral student, Mladen Berić, should be appointed as assistant professor of Theoretical Mathematics [1, p. 247]. Other students of Mihailo Petrović followed the same or a similar path, and Belgrade Mathematical School has lasted so far, even though there is a problem of its identity and, in particular, authority.

However, Mihailo Petrović significantly influenced not only the development of scientific, but also teaching staff in the field of mathematics. In that sphere, since 1897, he was occasionally engaged as a supervisor of mathematics teaching in secondary schools and an envoy for the maturity exams both in Belgrade grammar schools and grammar schools in the interior of Serbia. He was the president of professorship exam commission almost his whole working life. For a while he was a member, and in 1912 he became the President of the Educational Council of the Ministry of Education of Serbia. He reviewed high-school textbooks and he was always aware of what was going on in secondary schools and what quality of textbooks was used.

"Mihailo Petrović devoted a huge amount of his time to teaching. For many years he was the only mathematics professor at the Faculty of Philosophy and the studies of mathematics lasted for four years; he held all mathematics courses exclusively. Understandably, in such circumstances not all mathematical disciplines could be represented or equally studied. He published authorised scripts for eight courses he was teaching, and even his students themselves published several un-authorised scripts based on the recorded lectures of his. Three coursebooks of his were published. He alone organized students' exercises and managed students' work on term papers. He provided the library with books and journals and set up two scientific journals for publishing scientific mathematical papers" [6].

Our great mathematician seriously dealt with concrete problems of mathematics teaching. Namely, at the Congress of Mathematicians in Rome (6–11th April 1908), a three-member commission was appointed with a task of forming the International Commission on Mathematical Instruction, ICMI. The Commission was formed in Cologne, in September of that same year, and it consisted of 43

members from 25 countries from Europe, America and Australia. The Serbian delegate in the Commission was Mihailo Petrović. In his report (see [2], Vol. 10: Articles, Studies, pp. 84–92) which was published in 1913, Mihailo Petrović wrote about two meetings of ICMI: in Milan (18–21 September 1911) and in Cambridge (August 1912). At the gathering in Cambridge, Petrović presented his work: *Fonctions implicites oscillantes*. The conclusions of those two gatherings presided by Felix Klein were that mathematics should be taught according to the reformed programme which had been adopted in Meran in Italy (the so-called Meran programme) in 1905, and the teaching should be based on intuition and obviousness, while constantly gathering and interchanging teaching experiences from different countries.

Mihailo Petrović was the author of very interesting texts for secondary school students. Those texts deal with interesting problems (eye deceptiveness, wrong conclusions and badly drawn sketches ...), but they also explain, in a popular and intelligible way, some very complex problems (indefinite, impossible, incompletely defined tasks, stereometric inequalities, squaring the circle, angle trisection ...) (see [2], Vol. 10: Articles, Studies, pp. 15-63).

Furthermore, Mihailo Petrović was the Vice-dean several times, and he was the Dean of the Faculty of Philosophy in Belgrade from 1908 until 1910. In the year 1927 he was elected to be the rector of Belgrade University, however, he refused to accept that significant position having twice been nominated by academicians for the position of the president of SANU (Serbian Academy of Science and Arts) but rejected by the higher authorities of that time on the grounds of his long-lasting and true friendship with Đorđe Karađorđević, Crown Prince of Serbia.

5. Other contributions of Mihailo Petrović

Mihailo Petrović had an interesting personality because beside mathematics and its applications he dealt with many other things. He wrote bylaws, papers and reports on many scientific gatherings; he invented several successful patents; he was the author of pedagogical mathematical articles for elementary and secondary schools; he was a representative of mechanical reasoning in natural philosophy; an astronomer, a correspondent to daily newspapers; noted between the two wars as the writer of travelogues in our literature, a historian, an essay writer, a creator of very successful systems of cryptography for the needs of the army and diplomacy; the collector of traditional melodies and folklore; the author of professional texts in the fields of fishing and economics, an oceanographer and seafarer, etc.

Enormous scientific achievements prove him to have been a diligent, systematic and disciplined man of enormous energy no matter what scientific fields beside mathematics are concerned. The best proof of his scientific curiosity, versatility and broad education are definitely his philosophical, literary and other works. His books "Mathematical Phenomenology" and "Metaphors and Allegories", which connect mathematics with other fields, as well as his novel "The Eel" and travel books describe a man who did everything in his life with maximum care, professionally and expertly.
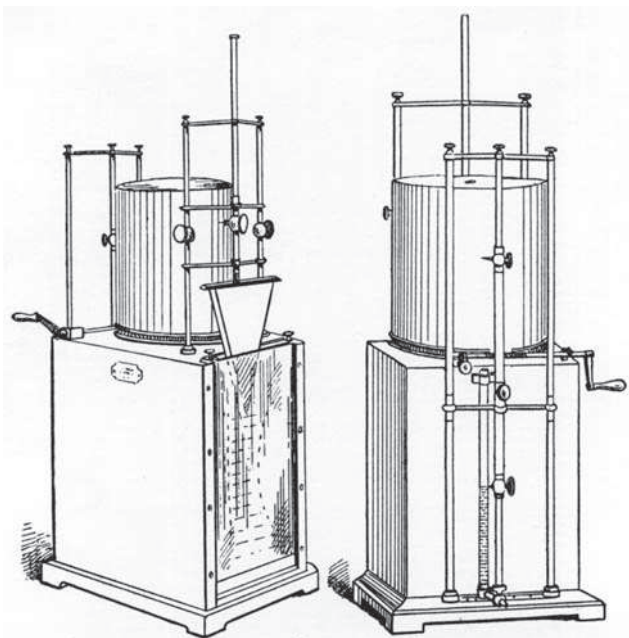
Mihaila Petrović's hydro-integrator for which he was awarded gold medal
at the World Exhibition in Paris in 1900

As a person, he was a favourite among the people he socialized with, and he socialized with all kinds of people, ordinary people and the Danube fishermen, his friends from the musical society "Suz", Belgrade University professors, academicians and the greatest mathematicians of the world of his time.

The collected works of Mihailo Petrović [2] – a capital work published in 15 volumes at the end of the last century by the Institute for Textbook Publishing and Teaching Aids, are the best proof of the comprehensiveness of his interests and his extraordinary scientific talent. Therefore, studying the biography of Mihailo Petrović and his collected works can be instructive for every Serbian citizen since it presents an interesting example of the achievements of a man who appreciated knowledge, who studied and created during all his life, working in favour of his family, his friends, his profession and his Serbia, trying to make life in it better, easier and wealthier by his own personal example and contribution.

6. Conclusion

It is certain that Mihailo Petrović's scientific and pedagogical work significantly enriched the scientific and pedagogical life in the field of mathematics in Serbia of his time and crucially influenced the development of mathematical sciences and teaching of mathematics in Serbia at the end of the 19th and in the first half of the 20th century.

The study of real contribution of Mihailo Petrović to both Serbian and world science asks for additional and more detailed research of his ideas, as well as studying the implications of Mihailo Petrović's works on achievements and development of mathematical and other sciences so far.

In order to summarize the scientific and pedagogical contribution of Mihailo Petrović to Serbian and world science, let us quote the academician Miodrag Tomić:

"Mihailo Petrović is a significant personality not only of our scientific but also cultural history at the end of the 19th and in the first half of the 20th century. His life and his work left a visible trace on generations of our mathematicians at Belgrade University. His gift, efforts and success made our mathematical science cross the borders of our country. If the first steps in science are the most difficult ones, they are also the most significant ones. He ignited the flame that even the wars could not extinguish. His example was followed by many students of his and that is what makes scientific progress, and that invisible contribution of Petrović's is as significant as his work".

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